Online Appendix of "Consistency As A Means to Comparability: Theory and Evidence"

1 Micro-foundation of the standard setter's objective function

We assume that in setting consistency, $\{m^j\}_{j=1}^N$, the standard setter maximizes the aggregate informativeness of firms' reports $\left\{r_1^j, r_2^j\right\}_{j=1}^N$ by minimizing the aggregate conditional variance of firms' cash flows, V_i):

$$\{m^{j*}\}_{j=1}^N = \arg\min_{\{m^j\}_{j=1}^N} \sum_{i=1}^{2} var\left(V_i | \left\{r_1^j, r_2^j, m^j\right\}_{j=1}^N\right). \quad (1)$$

This function can be micro-founded in a number of ways. For example, consider a setting in which a risk-neutral investor chooses an investment decision k_i to maximize his expected payoffs $E\left[k_i V_i - \frac{k_i^2}{2} | \left\{r_1^j, r_2^j, m^j\right\}_{j=1}^N\right]$, after observing the two firms' reports $\left\{r_1^j, r_2^j, m^j\right\}_{j=1}^N$. In this payoff function, $k_i V_i$ captures his benefits from making k_i units of investment in firm i, and $\frac{k_i^2}{2}$ represents his investment costs. The investment costs could be related to his capital raising in an imperfectly competitive capital market. In such a market, the supply curve of capital is increasing in the cost of capital, which makes the marginal cost of capital increasing in the amount that the investor borrows. As a result, the investment cost is convex. Because the investor cannot observe the realization of firms' fundamental cash flows, he benefits from reports with higher informativeness (or lower conditional variance, i.e., $var(V_i|\left\{r_1^j, r_2^j, m^j\right\}_{j=1}^N))$. After taking the expectation of the investor's objective function, his ex ante investment payoffs reduce into $\frac{\bar{V}_i^2+\sigma_{V_i}^2-var(V_i|\left\{r_1^j,r_2^j,m^j\right\}_{j=1}^N)}{2}$, which strictly decrease with $var(V_i|\left\{r_1^j, r_2^j, m^j\right\}_{j=1}^N)$. Thus, the standard setter maximizes the investor's welfare by minimizing the sum of conditional variances. Alternatively, one can assume that the investor is risk-averse with a constant absolute risk aversion (CARA) utility, $E\left[-e^{-\tau k_i V_i}|\left\{r_1^j, r_2^j, m^j\right\}_{j=1}^N\right]$. Since all random variables in our model are normally distributed, the investor's expected payoffs equal $E\left[V_i|\left\{r_1^j, r_2^j, m^j\right\}_{j=1}^N\right]k_i - \frac{\tau}{2}k_i^2 var\left(V_i|\left\{r_1^j, r_2^j, m^j\right\}_{j=1}^N\right)$, which can be reduced into $\frac{1}{2\tau}\left(\frac{\bar{V}_i^2+\sigma_{V_i}^2}{var\left(V_i|\left\{r_1^j,r_2^j,m^j\right\}_{j=1}^N\right)}-1\right)$ and also strictly decrease with $var\left(V_i|\left\{r_1^j, r_2^j, m^j\right\}_{j=1}^N\right)$.

2 Firm level consistency-based comparability analysis

In this section, we derive the firm level DKV measure, firm level fundamental volatility, and firm level fundamental correlation. We then assess whether the DKV measure can continue to serve as a reasonable proxy for consistency-based comparability at the firm level, and if so, under what conditions. Further, we check whether there exist clear theoretical predictions about the correlation between comparability and fundamental volatility/correlation at the firm level.

Building on the structure laid out in Section 3.1 of the paper, we derive the firm level DKV measure as follows:

$$r_1 = f_1(V_1) = V_1 + \sum_{j=1}^N x_1^j \varepsilon_1^j, \tag{2}$$

$$r_2 = f_2(V_2) = V_2 + \sum_{j=1}^N x_2^j \varepsilon_2^j. \tag{3}$$

Substituting V_1, the terminal cash flows of firm 1, into equation (2) gives:

$$r_1' = f_2(V_1) = V_1 + \sum_{j=1}^{N} x_2^j \varepsilon_2^j. \tag{4}$$

We can then write the firm level DKV measure as:

$$CB\_COMP = -E(r_1' - r_1)^2 = -E\left(\sum_{j=1}^{N} x_2^j \varepsilon_2^j - \sum_{j=1}^{N} x_1^j \varepsilon_1^j\right)^2. \tag{5}$$

Substituting the optimal level of consistency $m^j = m^{j*}$ into equation (5), we simplify CB_COMP as:

$$\begin{aligned}
CB_COMP &= -E\left(\sum_{j=1}^{N} x_2^j \varepsilon_2^j - \sum_{j=1}^{N} x_1^j \varepsilon_1^j\right)^2 \qquad (6)\\
&= -E\left(\sum_{j=1}^{N} x_2^j \left(m^{j*}\delta^j + \left(1-m^{j*}\right)\eta_2^j\right) - \sum_{j=1}^{N} x_1^j \left(m^{j*}\delta^j + \left(1-m^{j*}\right)\eta_1^j\right)\right)^2\\
&= -E\left(\sum_{j=1}^{N}\left[\left(x_2^j - x_1^j\right)m^{j*}\delta^j + \left(x_2^j\eta_2^j - x_1^j\eta_1^j\right)\left(1-m^{j*}\right)\right]\right)^2\\
&= -\sum_{j=1}^{N}\left\{\left(x_2^j - x_1^j\right)^2\left(\sigma_\delta^j\right)^2\left(m^{j*}\right)^2 + \left[\left(x_2^j\right)^2 + \left(x_1^j\right)^2\right]\left(\sigma_\eta^j\right)^2\left(1-m^{j*}\right)^2\right\}.
\end{aligned}$$

Note that

$$\begin{aligned}
\frac{\partial CB_COMP}{\partial m^{j*}} &= -2\left\{\left(x_2^j - x_1^j\right)^2\left(\sigma_\delta^j\right)^2 m^{j*} - \left[\left(x_2^j\right)^2 + \left(x_1^j\right)^2\right]\left(\sigma_\eta^j\right)^2\left(1-m^{j*}\right)\right\} \qquad (7)\\
&= -2\left\{\left[\left(x_2^j - x_1^j\right)^2\left(\sigma_\delta^j\right)^2 + \left[\left(x_2^j\right)^2 + \left(x_1^j\right)^2\right]\left(\sigma_\eta^j\right)^2\right]m^{j*} - \left[\left(x_2^j\right)^2 + \left(x_1^j\right)^2\right]\left(\sigma_\eta^j\right)^2\right\}\\
&\propto -\left\{m^{j*} - \frac{\left(\sigma_\eta^j\right)^2}{\frac{\left(x_2^j - x_1^j\right)^2}{\left(x_2^j\right)^2 + \left(x_1^j\right)^2}\left(\sigma_\delta^j\right)^2 + \left(\sigma_\eta^j\right)^2}\right\},
\end{aligned}$$

where "$\propto$" means "having the same sign with." Accordingly, $\frac{\partial CB\_COMP}{\partial m^{j*}} > 0$ iff

$$m^{j*} < \frac{\left(\sigma_{\eta}^{j}\right)^{2}}{\frac{\left(x_{2}^{j}-x_{1}^{j}\right)^{2}}{\left(x_{2}^{j}\right)^{2}+\left(x_{1}^{j}\right)^{2}}\left(\sigma_{\delta}^{j}\right)^{2}+\left(\sigma_{\eta}^{j}\right)^{2}}, \tag{8}$$

which holds if $\left(x_{2}^{j}-x_{1}^{j}\right)^{2}$ is sufficiently small. That is, CB_COMP is a good measure of m^{j*} when the two firms' portfolios of transactions $\left\{x_{i}^{j}\right\}_{j=1}^{N}$ are sufficiently close.

Turning to fundamental volatility and correlation at the firm level, recall that, the total cash flows to firm i in our model are simply the sum of its cash flows from all transactions:

$$V_{i} = \sum_{j=1}^{N} x_{i}^{j} v_{i}^{j}. \tag{9}$$

We can thus calculate the fundamental volatility of firm i's total cash flows as:

$$\sigma_{V_i}^{2} = var\left(\sum_{j=1}^{N} x_{i}^{j} v_{i}^{j}\right) = \sum_{j=1}^{N}\left(x_{i}^{j}\right)^{2}\left(\sigma_{v}^{j}\right)^{2}. \tag{10}$$

We can also calculate the fundamental correlation between firms' total cash flows as:

$$\rho_{V} \equiv \frac{\sigma_{V_1 V_2}}{\sqrt{\sigma_{V_1}^{2}}\sqrt{\sigma_{V_2}^{2}}} = \frac{\sum_{j=1}^{N} x_{1}^{j} x_{2}^{j} \rho_{v}^{j}\left(\sigma_{v}^{j}\right)^{2}}{\sqrt{\sum_{j=1}^{N}\left(x_{1}^{j}\right)^{2}\left(\sigma_{v}^{j}\right)^{2}}\sqrt{\sum_{j=1}^{N}\left(x_{2}^{j}\right)^{2}\left(\sigma_{v}^{j}\right)^{2}}}. \tag{11}$$

From equations (10) and (11), we can see that firm level volatility $\sigma_{V_i}^{2}$ and firm level correlation ρ_V have complicated mathematical expressions because they are affected by not only transaction specific but also inter-transaction characteristics (e.g., how x_1^j compares with x_2^j). Firm level DKV measure CB_COMP is similarly affected by x_i^j (equation (6)). Thus, without making additional assumptions on x_i^j, our model gives no clear theoretical predictions about the correlation between firm level CB_COMP and $\sigma_{V_i}^{2}$ and the correlation between firm level CB_COMP and ρ_V.